On the locality of the pseudovariety $DG$

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Abstract

With every pseudovariety $V$ of finite monoids, two pseudovarieties of finite categories are naturally associated. The first one is the pseudovariety $gV$ generated by $V$, and the second one is the pseudovariety $lV$ consisting of all finite categories all of whose local monoids belong to $V$. Of course, $gV$ is a subclass of $lV$. The pseudovariety $V$ of finite monoids is said to be local if $gV = lV$ holds.

Many important pseudovarieties of finite monoids have been shown to be local in the past. This concerns, in particular, the pseudovariety $DS$ of all finite monoids all of whose regular $D$-classes are subsemigroups, the pseudovariety $DO$ of all finite monoids all of whose regular $D$-classes are orthodox subsemigroups, and the pseudovariety $DA$ of all finite monoids all of whose regular $D$-classes are aperiodic subsemigroups. On the other hand, it is well known that the the pseudovariety $J$ of all finite $J$-trivial monoids is not local.

The question whether the pseudovariety $DG$ of all finite monoids all of whose regular $D$-classes are subgroups is local, which has remained open until now, thus became of particular interest. It is the purpose of this talk to announce an affirmative answer to this question. Other results of this kind are also obtained. Thus for every non-trivial extension closed pseudovarieties $H$ of finite groups, the pseudovariety $DH$ consisting of all finite monoids all of whose regular $D$-classes are groups from $H$ is also local. For instance, the pseudovariety $DG_{sol}$ of all finite monoids all of whose regular $D$-classes are solvable groups turns out to be local in this way. But it is still not everything that can thus be obtained. For example, the pseudovariety $DG_{nil}$ of all finite monoids all of whose regular $D$-classes are nilpotent groups is local, as well. It should be noted that the approach applied to gain these results is, in fact, quite elementary, but, on the other hand, it is rather laborious.