Contents

Invited Speakers

Finitely presented modules over Leavitt algebras
Pere Ara
Universitat Autonoma de Barcelona, Spain
para@mat.uab.es

Given a field k and a positive integer n, we study the structure of the finitely presented modules over the Leavitt k-algebra L of type $(1, n)$, which is the k-algebra with a universal isomorphism $i : L \to L^{n+1}$. The abelian category of finitely presented left L-modules of finite length is shown to be equivalent to two different subcategories of finitely presented modules over the free algebra of rank $n + 1$. This allows us to use Schofield's exact sequence for universal localization to compute the K_1 group of a certain von Neumann regular algebra of fractions of L.

Hochschild cohomology of algebras and epimorphisms
José Antonio de la Peña
UNAM, México
jap@matem.unam.mx
Coauthors: Changchang Xi

Let $\varphi : A \to B$ be a homological epimorphism of k-algebras. We investigate the relationship of the Hochschild cohomologies $H^i(A)$ and $H^i(B)$ of A and B, and show that they can be connected by a long exact sequence. In particular, if A is a quasi-hereditary ideal, then the long exact sequence provides information on $H^i(A)$, $H^i(B)$ and the extension groups between costandard modules and standard modules, thus one can actually computer $H^i(A)$ inductively. As a consequence, we obtain the Hochschild cohomology of all non-semisimple Temperley-Lieb algebras and representation-finite Schur algebras.

Rational Cherednik algebras and applications
Iain Gordon
University of Glasgow, UK
ig@maths.gla.ac.uk

We will present some recent work on the representation theory of rational Cherednik algebras (introduced by Etingof and Ginzburg) and its application to questions in algebraic combinatorics and Hilbert schemes of points on the plane.

Left Cotorsion Rings

Pedro A. Guil Asensio
University of Murcia, Spain
paguil@um.es
Coauthors: Ivo Herzog

Let R be an associative ring with identity. A left R-module M is called cotorsion if $Ext^1_R(F, M) = 0$ for every flat left R-module F. Cotorsion modules are a generalization of pure-injective modules and their interest derives from the recent result of Bican, El Bachir and Enochs that every module admits a cotorsion envelope.

We prove that if R is an associative ring which is cotorsion as a left module over itself, and J is the Jacobson radical of R, then the quotient ring R/J is a left self-injective von Neumann regular ring and idempotents lift modulo J. In particular, if R is indecomposable, then it is a local ring.

Representation dimension of artin algebras

Osamu Iyama
Himeji Institute of Technology, Japan
iyama@sci.himeji-tech.ac.jp

30 years ago, M. Auslander introduced a concept of representation dimension of artin algebras, which measures homologically how far an artin algebra is from being of finite representation type. His methods have been effectively applied not only for the representation theory of artin algebras, but also for the theory of quasi-hereditary algebras of Cline-Parshall-Scott by Dlab-Ringel. Recently, the author proved that any artin algebra has a finite representation dimension by showing that any module is a direct summand of some module whose endomorphism ring is quasi-hereditary. Our method is to construct certain chain of subcategories of mod Λ. It was also applied to solve Solomon's second conjecture on zeta functions of orders by the author. We will formulate it in terms of rejective subcategories, which was applied to study the representation theory of orders and give a characterization of their finite Auslander-Reiten quivers.

Comparing Schur algebras, upwards and downwards

Steffen Koenig
University of Leicester, UK
sck5@mcs.le.ac.uk
Coauthors: Ming Fang and Anne Henke

Fix an infinite field k of arbitrary characteristic and let G be the general linear group $GL(n,k)$. The polynomial representations of G split into homogeneous components, and the homogeneous representations of degree r are modules over a finite dimensional associative algebra, the Schur algebra $S(n,r)$. I will explain methods of comparing Schur algebras in different degrees, and the consequences of such structural comparisons on the numerical level of decomposition numbers.

Corner Ring Theory: A Generalization of Peirce Decompositions

Tsit-Yuen Lam
University of California, Berkeley, USA
lam@Math.Berkeley.EDU

One of the main tools in classical ring theory is the Peirce decomposition of a ring R with respect to a pair of complementary idempotents e and f. In particular, the "Peirce corner" eRe of a ring R has always been a popular topic for research. In this talk, we'll present a general theory of corner rings in a noncommutative ring R that generalizes the classical Peirce decompositions of R. Descent properties and the multiplicative structure of corner rings will be discussed.

Representation type of Commutative Noetherian Rings

Lawrence Levy
University of Nebraska and University of Wisconsin, USA
levy@math.wisc.edu
Coauthors: Lee Klingler

The purpose of this project is to extend the fundamental theorem of abelian groups to a structure theory for finitely generated modules — including their direct-sum relations — over as many commutative noetherian rings R as possible. When we succeed we call R-mod "tame", and when we fail we prove that it is wild. In 1911 Steinitz proved that Dedekind domains are tame, and as far as we know, no other noetherian *domains* have been been proved to be tame, between then and the present project, although some local non-domains were proved tame by Nazarova and Roiter in 1969, and artinian principal ideal rings have been long known to be tame.

We prove that the class of tame (commutative noetherian) rings is a narrow class of rings of Krull dimension ≤ 1 properly containing the above-mentioned rings. In our description of direct-sum behavior, Steinitz's ideal class group is replaced by a possibly infinite family of genus class groups and the web of homomorphisms between them. Direct-sum cancellation holds if and only if all of these homomorphisms are one-to-one. (There is a possible exception to our tameness results, involving characteristic 2. But this exception never arises when R is a ring of algebraic integers or an algebra over a field of characteristic not= 2.)

Connection on stable bundle

Olivier Mathieu
Université de Lyon, France
`mathieu@igd.univ-lyon1.fr`

Let X be a Riemmann surface, i.e a genus g surface with a complex structure.

Definition. An holomorphic vector bundle E is called *stable* if $c(E) = 0$ but $c(F) < 0$, for any proper subbundle F.

Here "stable" stands for stable of slope 0. The notation $c(F)$ is the first Chern class of F. The meaning of "$c(F) < 0$" is explained by the natural identification of the second cohomology group of X with Z.

Narashiman and Seshadri have shown that a stable bundle admits a unique hermitian holomorphic connection. When X and E come from an algebraic curve and an holomorphic bundle defined over a number field K, the previous theorem attach to each infinite place of K a certain connection.

We show a similar statement for finite places of K. Then we state a conjecture about the algebraicity of solutions of certain differential equations.

Quadratic algebras of skew type

Jan Okninski
Warsaw University, Poland
`okninski@mimuw.edu.pl`

We consider finitely generated unitary algebras $A = K\langle x_1, \ldots, x_n : R\rangle$ over a field K, defined by a system R of relations of the form $u = v$ or $w = 0$, where u, v, w are words in the generators x_i. Our main interest is in noetherian algebras of this type.

Certain necessary and sufficient conditions for A to be noetherian are presented. Then one of the goals is to see how the homogeneous information, that is, structural and combinatorial information on the semigroup defined by the same presentation, determines the properties of the algebra. In this context, the PI-property, the Gelfand-Kirillov dimension and the classical Krull dimension, the prime radical and the prime ideals of the algebra are considered.

A special class of interest consists of algebras defined by square-free relations of the form $u = v$, where u, v are words of length 2. One of the motivations comes from problems concerning set theoretical solutions of the Yang-Baxter equation.

Quasideterminants and Roots of Polynomials over Division Rings

Barbara L. Osofsky
Rutgers University, New Brunswick, USA
osofsky@math.rutgers.edu

Except for very special cases such as the quarternions $\mathbb{H}$, it is difficult to say anything significant about square matrices over a noncommutative division ring. For example, there is a division ring $\mathcal{F}(X)$ generated over the rationals $\mathbb{Q}$ by the free semigroup on a set X of noncommuting indeterminants, and it seems as if it should be very hard to say anything at all about matrices over it. I. Gelfand, V. Retakh, and others have developed a theory of quasideterminants to replace using determinants when one is looking at square matrices over noncommutative rings. In the case of matrices over division rings, these quasideterminants are the last pivots in doing gaussian elimination to invert the matrix. Over any division ring Gelfand and Retakh compute the quasideterminant of a van der Monde matrix obtained by pivoting on the diagonal when the matrix is invertible. We present some of their theory and applications to looking at the right roots of polynomials whose coefficients lie in a division ring. Let $p(x)$ be a monic polynomial $p(x) = \sum_{i=0}^{n} a_i x^i$ of degree n in an indeterminant x that need not commute with all coefficients of $p(x)$. If $p(x)$ has n roots independent in the sense that an appropriate van der Monde matrix is invertible, then the coefficients must be rational functions of these roots, and the rational functions involved play a role in the study of symmetric functions very close to the role played by the elementary symmetric functions in the commutative case. There is an algebra Q_n with a very rich structure associated with all such p, and we present some of this structure.

Basic properties of the module category of an artin algebra.

Claus Michael Ringel
University of Bielefeld, Germany
ringel@mathematik.uni-bielefeld.de

The lecture will concentrate on quite old, but also very new investigations concerning the Brauer-Thrall conjectures, outlining considerations which complement the usual Auslander-Reiten methods. In particular, I will focus the attention to the Gabriel-Roiter measure which was introduced (as "Roiter measure") by Gabriel in order to make explicit the combinatorial scheme of Roiter's proof of the first Brauer-Thrall conjecture. This proof which was intended just for categories of bounded representation type gives a lot of insight for artin algebras of arbitrary representation type.

Goldie dimension applied to Linear Operator Theory

Catarina Santa-Clara
University of Lisbon, Portugal
cgomes@ptmat.fc.ul.pt
Coauthors: Laura Iglésias, Fernando C. Silva

There exist striking analogies between the eigenvalues of Hermitian compact operators, the singular values of compact operators and the invariant factors of homomorphisms of modules over principal ideal domains, namely: diagonalization theorems, interlacing inequalities and Courant–Fisher type formulas.

We unify these results using Goldie dimension of modular lattices (a generalization of Goldie dimension of modules).

Categorical invariance of automorphism groups of rings and algebras

Manuel Saorín
Universidad de Murcia, Spain
msaorinc@um.es

The first part of the talk will be dedicated to a summary of results on the categorical invariance of groups of automorphisms of algebras. We shall specially concentrate on the group of outer automorphisms, denoted $Out(A)$, which has received a lot of attention in recent times. When A is finite dimensional, $Out(A)$ is an algebraic group, the identity component of which we denote by $O(A)$. We shall mention results by Brauer, Pollack, F. Guil Asensio, Huisgen-Zimmermann (the last two in collaboration with the author) and Rouquier which led, in case K is algebraically closed, from checking that $O(A)$ is invariant under Morita equivalences to checking its invariance under derived equivalences and, in case A is self-injective, under stable equivalences of Morita type.

In the second part of the exposition, we will adopt the more general setting of rings, or even algebras over an arbitrary commutative ground ring K. We will show previously unknown groups of automorphisms of algebras which are invariant under Morita equivalences. For instance, in case A is a semiregular K-algebra, we consider the group H of automorphisms f of A satisfying the property that, for every idempotent e in A, there exist a in $eAf(e)$ and b in $f(e)Ae$ such that $ab = e$, $ba = f(e)$. We shall show that $H/Inn(A)$ is Morita invariant. Likewise, for an arbitrary K-algebra, we shall show that if $Aut(A)_1$ is the group of automorphisms of A inducing the identity modulo $J(A)$ (the Jacobson radical of A), then the image of $Aut(A)_1$ by the canonical group homomorphism $Aut(A) \longrightarrow Out(A)$ is Morita invariant. Finally, we will discuss the Morita invariance of the group $Aut(A)_1/Inn^*(A)$, where $Inn^*(A)$ is the subgroup of inner automorphisms induced by units of the form $1 - x$, with x in $J(A)$.

A decomposition theory for irreducible components of module varieties

Jan Schröer
University of Leeds, UK
jschroer@maths.leeds.ac.uk
Coauthors: William Crawley-Boevey and Christof Geiss

We present a decomposition theory for irreducible components of the varieties of modules over a finitely generated algebra. Applied to preprojective algebras, this can be used to get a better understanding of the dual canonical basis of quantum groups.

Cotilting modules versus Canonical modules for Cohen-Macaulay rings

Alberto Tonolo
Universitá di Padova, Italy
tonolo@math.unipd.it

Let R and S be left and right Noetherian rings, respectively. We say that a faithfully balanced bimodule ${}_RV_S$ is *N-partial cotilting* (N stays for Noetherian) *of injective dimension less or equal than* n if it satisfies the following properties:

N1: $id_R V$ and $id V_S$ are less or equal n.

N2: $Ext^i_R(V, V) = 0$ and $Ext^i_S(V, V) = 0$ for each i greater or equal than 1.

N3: ${}_RV$ and V_S are finitely generated.

These are exactly the modules considered by Miyashita in [2] in the section dedicated to dualities. Denoted by R-mod and $mod-S$ the categories of finitely generated left R- and right S- modules, let us consider the subclasses

$\mathcal{E}_e({}_RV) = \{M \in R - mod : Ext^i_R(M, V) = 0$, for all i different from $e\}$ and

$\mathcal{E}_e(V_S) = \{N \in mod - S : Ext^i_S(N, V) = 0$, for all i different from $e\}$.

We will denote by $\mathcal{E}_e$ both these classes. Miyashita proved (see [2, Theorem 6.1]) that for each module $L \in \mathcal{E}_e$

(i) $Ext^e(L, V)$ belongs to $\mathcal{E}_e$, and

(ii) L is isomorphic to $Ext^e(Ext^e(L, V), V)$.

Let A be a graded Gorenstein algebra of Krull dimension n. The regular bimodule ${}_AA_A$ is faithfully balanced and it satisfies properties $(N1-3)$: therefore it is a N-partial cotilting bimodule of injective dimension n. The class $\mathcal{E}_e({}_AA_A)$ coincides with the family of finitely generated modules which are either 0 or Cohen-Macaulay of dimension $n - e$. Thus, the Miyashita's Theorem generalizes well known results (see [1, Theorem 3.3.10]) on the dualities induced by the *canonical module.*

We present some new analogies between these two theories. In particular, we will concentrate on generalizations to the cotilting setting of the characterizations of *sequentially Cohen-Macaulay* modules [3, Definition III.2.9] and of Cohen-Macaulay rings admitting a canonical module (see [1, Theorem 3.3.7]).

References

[1] W. Bruns and J. Herzog. *Cohen-Macaulay rings*, Cambridge studies in advanced mathematics **39**, Cambridge University Press, 1993.

[2] Y. Miyashita. *Tilting modules of finite projective dimension*, Math. Z. **193**, (1986), 113–146.

[3] R. P. Stanley. *Combinatorics and Commutative Algebra*, 2nd ed., Progress in Mathematics **41**, Boston, Birkhäuser, 1996.

Cotorsion pairs

Jan Trlifaj
Univerzita Karlova, Praha
trlifaj@karlin.mff.cuni.cz

Cotorsion pairs are analogs of (non-hereditary) torsion pairs with Hom replaced by Ext. They naturally occur in approximation theory of modules. In the first part on my talk, I will recall the basic relations between approximations and cotorsion pairs. In the second part, I will concentrate on tilting and cotilting approximations. By a result of Angeleri and Coelho, if $C = (A, B)$ is a hereditary cotorsion pair of finite type with $A \subset P$, then C is a tilting cotorsion pair. The converse implication remains open. I will present my recent joint work with Eklof showing that the converse holds in the particular case of Dedekind domains, and that all 1-tilting cotorsion pairs over ω-noetherian rings are of countable type.

Flops and derived equivalences.

Michel Van den Bergh
Limburgs Universitair Centrum, Diepenbeek, Belgium
vdbergh@luc.ac.be

Recently Bridgeland proved that all crepant resolutions of a three dimensional Gorenstein singularity are derived equivalent. We will show how this result can be understood using non-commutative ring theory.

Representation Theory of Algebras

Some coreflective categories of topological modules
Daciana Alina Alb
University of Oradea, Romania
dalb@uoradea.ro

Let R be a topological ring with identity and ${}_RM$ the category of all unitary topological left R-modules. We give conditions on a ring R under which the subcategory ${}_RM$ of modules whose underlying space is a P-space is coreflective.

Theorem Let R be a fixed locally σ-compact ring. Then the subcategory ${}_RM$ of all P-modules is coreflective.

Remark If R is a topological ring whose topological space is a P-space, then the category ${}_RM$ of P-modules is coreflective.

Definition Let m be an infinite cardinal. We will say that a topological space (X, T) is a m-space provided the intersection of any family of open subsets of cardinality $< m$ is open.

Theorem Let R be a topological ring and $h(R) = m$. Then the category ${}_RM$ of all topological R-modules whose topology is a m-topology is coreflective in the category of all topological R-modules.

Acknowledgment The author is grateful to professors Mitrofan Cioban and Mihail Ursul for their constant interest and helpful suggestions in this work.

References

[1] L. Gillman, M. Jerisson, Rings of Continuous Functions, Van Nostrand, 1960, p. 62-63

Quadratic Malcev superalgebras
Helena Albuquerque
Universidade de Coimbra, Portugal
lena@mat.uc.pt
Coauthors: Said Benayadi

A quadratic Malcev superalgebra is a Malcev superalgebra with a non-degenerate supersymmetric even invariant bilinear form B; B is called an invariant scalar product. In this talk, we present the inductive classifications of quadratic Malcev algebras and of Malcev superalgebras such that the even part is a reductive Malcev algebra and the action of the even part on the odd part is completely reducible.

Classification of integral table algebras via a given subset of their algebra constants

Zvi Arad
Netanya Academic College, Netanya, Israel
aradtzvi@macs.biu.ac.il
Coauthors: Y. Erez, M. Muzychuk

The concept of "table algebras" was defined by Arad and Blau in order to study, in a uniform way, products of conjugacy classes and irreducible characters of a finite group. Since then the theory of table algebras was deeply developed by many authors among them Z. Arad, H. Arisha, H. Blau, F. Bunger, D. Chillag, M.R. Darafsheh, J. Erez, E. Fisman, V. Miloslavsky, M. Muzychuk, A.Rahnamai, C. Scopolla and B. Xu. We refer the reader to the book Z.Arad, and M. Muzychuk, *Standard integral table algebra generated by a nonreal element of small degree*, Lecture Notes in Mathematics, Springer, vol. 1773 (2002), pp. 1126, where recent progress in this area is presented. Table algebras are strongly connected to various concepts in modern algebra developments in the last century. Let us mention the following well known algebras; Schur Rings defined by I.Schur in 1933; Calgebra defined by G.Hoheisel 1939 and Y.Kawada 1942; Association Schemes; BoseMesner algebras; Frobenius algebras; Hecke algebras; Distance Regular Graphs; Fusion Rings; Pseudo Groups; Products of conjugacy classes and Products of irreducible ordinary characters in finite group. New results on Table algebras yield new results for the above concepts.

Throughout C denotes the complex numbers, R the reals, R^+ the positive reals, Z^+ the positive integers, and N the natural numbers. Table algebras are a family of algebras with distinguished bases defined as follows:

Definition 0.1. Let $B = b_1, b_2, \ldots, b_r$ be a basis of finite dimensional, associative and commutative algebra A over the complex field C, with identity element $1_A = b_1$. Then (A, B) is a table algebra and B is a table basis if and only if the following hold:

1. For all i, j, m, $b_i b_j = \Sigma_{m=1}^{k} \lambda_{ijm} b_m$, with λ_{ijm} a nonnegative real number.
2. A has an algebra automorphism (denoted by $^-$) of order dividing 2, such that $b_i \in B$ implies that $\bar{b_i} \in B$. (Then $\bar{i}$ is defined by $b_i = \bar{b_i}$, and $b_i \in B$ is called real if $i = \bar{i}$.)
3. For all i, j, $\lambda_i j \neq 0$ if and only if $j = \bar{i}$.

By Arad and Fisman, there is an algebra homomorphism $f : A \to C$ such that $f(b_i) = f(\bar{b_i}) \in R^+$, for all i. Such a map f is uniquely determined by the orthogonality relations which hold for (A, B). The positive real numbers $f(b_i) = |b_i|$, $1 \leq i \leq r$, are called the degrees of (A, B).

A table algebra (A, B) is called integral iff all the structure constants λ_{ijm} and all the degrees $|b_i|$ are rational integers. A table algebra (A, B) is called homogeneous of degree λ iff $|B| > 1$ and, for some fixed $\lambda \in R^+$, degree $|b| = \lambda$. for all $b \in B^{\#}$, where $B^{\#} = B \setminus \{1\}$. A table algebra (A, B) is called standard iff $\lambda_{b\bar{b}1} = |b|$, for every element $b \in B^{\#}$.

The idea of using an algebra with a distinguished base to generalize properties of the group algebra goes back to Kawada, which followed after Hoheisel, which abstracted the duality be tween Products of conjugacy classes and Products of irreducible ordinary characters in finite group. Table algebras are also a kind of generalization of association schemes, studied inten sively by Bannai and Ito and strongly connected to the above mentioned concepts.

In our recent research we classify Integral Table Algebras via a given Subset of their Algebra Constants.

Endowild representation type and generic representations of finite posets

David M. Arnold
Baylor University, Waco, Texas, USA
David_Arnold@baylor.edu
Coauthors: Daniel Simson

The category of finite dimensional representations of a finite poset over an arbitrary field k is shown to have k-wild representation type if and only if it has k-endowild representation type. Representation type is characterized in terms of existence conditions for generic representations.

Lattice invariants, generic matrices and other rationality problems.

Esther Beneish
Central Michigan University, USA
beneile@cmich.edu

Let F be a field, let G be a finite group and let M be a ZG-lattice. Let $F(M)$ denote the quotient field of the group algebra of the abelian group M. The group G acts on $F(M)$ via its action on M. We study the following question: Is the fixed subfield of $F(M)$ under the action of G, stably rational over F? This question is connected to problems in various fields such as Brauer groups, geometric invariant theory, Noethers problem and generic matrices. We will discuss some of these connections and present our latest result on the generic division algebra,which is the following. Let p be a prime. We show that the center of the division algebra of pxp generic matrices over an algebraically closed field is stably isomorphic to a Kummer extension of a rational extension of the base field.

On simply connected generically finite algebras

Grzegorz Bobinski
Nicolaus Copernicus University, Torun, Poland
gregbob@mat.uni.torun.pl
Coauthors: Andrzej Skowronski

Algebras over algebraically closed fields can be divided according to complexity of their representation theory. The simplest case occurs if there exists only finite number of indecomposable finite dimensional modules. The next level in the hierarchy is occupied by generically finite algebras. The representation theory of strongly simply connected generically finite algebras is well understood. The main problem which appears in studying arbitrary generically finite algebras is connected with description of discrete modules. In the talk we present a class of simply connected generically finite algebras for which this description is possible. We also discuss the connection of representation type with the Jacobson radical of the module category.

Some classes of locally linearly compact generalized bounded algebras

Valeriu Bordan
Tiraspol State University, Chisinau, Republic of Moldova
Valeriu.Bordan@hotmail.ro

I introduce and study in this paper locally linearly compact algebras having neighborhoods of zero with special properties. The class of IN-algebras contain the class of all bounded locally linearly compact algebras. We give here necessary and sufficient conditions under which the semidirect product $AxA*$ is IN-algebra.

I consider only topological algebras over discrete field. All topological algebras are assumed to be Hausdorff and associative.

If A is a locally linearly compact topological vector F-space, then denote by $A*$ the dual space of A. By $AxA*$ is denoted semidirect product of A and $A*$.

Definition 1 A topological algebra A is called a LIN-algebra (RIN-algebra), if it contains an open linearly compact vector subspace V, such that $A \cdot V$ is included in V ($V \cdot A$ is included in V).

Definition 2 A Topological algebra A is called an IN-algebra, if it contains an open two-sided ideal V which is a linearly compact space.

Theorem For a locally linearly compact algebra A the following conditions are equivalent:

1. $AxA*$ is an IN-algebra;
2. $AxA*$ is a LIN-algebra;
3. $AxA*$ is a RIN-algebras;

4. A is an IN-algebra and there exist an open linearly compact subspace U of A such that $UU = \{0\}$

Tilting in tubes

Aslak Bakke Buan
Norwegian University of Technology and Science, Norway
aslakb@math.ntnu.no
Coauthors: Henning Krause

We classify tilting and cotilting modules for the completed path algebra of a quiver og type $\tilde{A}_n$ with linear orientation. The combinatorics of the collection of all tilting and cotilting modules is described in terms of Stasheff associahedra and a generalization of such.

On locally finite split Lie triple systems

Antonio J. Calderón Martín
University of Cádiz, Spain
ajesus.calderon@uca.es
Coauthors: Manuel Forero Piulestán

Lie triple systems appear as the natural ternary extension of Lie algebras. The classification in the finite-dimensional setup (over an algebraically closed field of characteristic zero) is well-known. In order to suggest a possible approach to a structure theory of infinite-dimensional Lie triple systems, we introduce and study split and locally finite Lie triple systems, stating that under certain conditions the standard embedding of a split Lie triple system is a split Lie algebra and that the standard embedding of a locally finite Lie triple system is a locally finite Lie algebra. We also give a description of certain locally finite simple split Lie triple systems.

Quantized universal enveloping algebras at the roots of unity and spherical conjugacy classes

Giovanna Carnovale
University of Padua, Italy
carnoval@math.unipd.it
Coauthors: Nicoletta Cantarini, Mauro Costantini

Let $U_\epsilon(\mathfrak{g})$ be the simply connected quantized enveloping algebra associated to a simple Lie algebra $\mathfrak{g}$ at the roots of unity. There exists a natural map from the set of irreducible representations of $U_\epsilon(\mathfrak{g})$ to the set of conjugacy classes of the simply connected algebraic group G with Lie algebra $\mathfrak{g}$. A conjecture of De Concini, Kac and Procesi relates the possible dimensions of irreducible representations of $U_\epsilon(\mathfrak{g})$ to the dimension of the corresponding conjugacy classes. We prove the De Concini, Kac and Procesi conjecture for representations associated to spherical conjugacy classes. The case of a Lie algebra of type A_n had been handled by N. Cantarini.

Local Theory of Almost Split Sequences for Comodules

William Chin
DePaul University, Chicago, Illinois, USA
wchin@condor.depaul.edu
Coauthors: Mark Kleiner, Declan Quinn

The authors have shown the existence of almost split sequences for comodules in some circumstances. In this talk, we discuss the relationship between almost split sequences for comodules and almost split sequences over finite dimensional subcoalgebras. We show that almost split sequences of comodules with a (quasifinitely copresented non-injective) finite dimensional comodule on the right are direct limits of almost split sequences of finite dimensional comodules. On the other hand, we construct certain sequences as limits of almost split sequences over finite dimensional subcoalgebras. Starting with a finite dimensional comodule M on the right, we construct these sequences (ending at M) in the category of quasifinite comodules as the direct limit of almost split sequences over finite dimensional subcoalgebras. These sequences satisfy a property in general weaker than being almost split. Starting with a finite dimensional comodule on the left, we construct sequences in the dual category to the category of quasifinite comodules as the inverse limit of almost split sequences over finite dimensional subcoalgebras. The sequences thus obtained turn out to be almost split in some circumstances.

Endomorphism algebras of projective modules over laura algebras

Flávio Ulhoa Coelho
Universidade de São Paulo, Brasil
`fucoelho@ime.usp.br`
Coauthors: Ibrahim Assem

Since its introduction by Happel and Ringel in the early eighties, the class of tilted algebras has been extensively studied in the representation theory of artin algebras. It is now considered to be one of the classes whose representation theory is best understood and most useful for the general theory. It was therefore natural to consider various generalisations of this notion. Thus, over the years, the following classes of algebras were defined and investigated: the quasi-tilted algebras by Happel-Reiten-Smalo (which generalise the tilted and the canonical algebras of Ringel), the shod algebras (which generalise the quasi-tilted) by Coelho-Lanzilotta, the weakly shod algebras (which generalise the shod and the representation-directed algebras) also by Coelho-Lanzilotta, the left and the right glued algebras (which generalise the tilted and the representation-finite algebras) by Assem-Coelho, and, finally, the laura algebras (which generalise all the previous classes) by Assem-Coelho (and independently by Reiten-Skowronski). We recall that an artin algebra A is said to be a laura algebra if all but at most finitely many non-isomorphic indecomposable A-modules are such that all its predecessors have projective dimension at most one or are such that all its successors have injective dimension at most one.

It was reasonable to ask the following question: if A is an artin algebra belonging to one of the classes above, and e is an idempotent in A, then does the endomorphism algebra eAe of the projective module eA belong also to the same class? The answer has already been shown to be positive for tilted algebras by Happel, for quasi-tilted algebras by Happel-Reiten-Smalo and for shod algebras by Kleiner-Skowronski-Zacharia. The objective of this talk is to show that it is positive as well for the remaining classes. Moreover, our proof yields also the quasi-tilted and the shod cases.

Tilting and cotilting-type modules

Gabriella D'Este
University of Milano, Italy
`Gabriella.Deste@mat.unimi.it`

We first compare more or less wide generalizations of the classical tilting modules (resp. cotilting modules) to modules of projective (resp. injective) dimension at most two. Next we investigate the gap between a cotilting-type module and its injective envelope.

Derived tame and wild algebras
Yuriy Drozd
Kyiv Taras Shevchenko University, Ukraine
yuriy@drozd.org

We prove that every finite dimensional algebra over an algebraically closed field is either derived tame or derived wild. We also consider the parameter number of complexes of prescribed rank over an algebra and show that it is upper semicontinuous in families of algebras. As a corollary, we get that the set of tame algebras of a fixed dimension is a countable union of open subsets of the variety of all algebras and every deformation of a tame algebra is tame.

The Jacobson radical of a Lie algebra
Joerg Feldvoss
Universitaet Hamburg, Germany
feldvoss@math.uni-hamburg.de

The Jacobson radical is important in the structure and representation theory of associative algebras. Besides a single occurrence in [M] an application of this concept to Lie algebras apparantly has not been considered in the literature. E. I. Marshall determines "his" Jacobson radical for arbitrary finite-dimensional Lie algebras in characteristic zero and finite-dimensional solvable Lie algebras in arbitrary characteristic. But in [M] there is neither a sytematic treatment nor a discussion of the representation-theoretic relevance. In my talk I will report on analogues of the Jacobson radical for (restricted) Lie algebras and their properties.

References

[M] E. I. Marshall: The Frattini subalgebra of a Lie algebra, J. London Math. Soc. 42 (1967), 416-422

Deformations of three dimensional Lie algebras
Alice Fialowski
Eotvos Lorand University, Budapest, Hungary
fialowsk@cs.elte.hu

I will consider versal deformations of three dimensional L(infinity) algebras, corresponding to ordinary Lie algebras. This approach allows as to characterize the moduli space of such Lie algebras. We use deformation theory as a guide to understand the picture.

Ringel duality for Harish-Chandra modules

André Fonseca
University of Leicester, UK and Universidade Lusófona, Portugal
avvf1@mcs.le.ac.uk

Since its definition by Bernstein, Gelfand and Gelfand in the seventies the category $\mathcal{O}$ has become a basic object of study admitting interesting generalizations in terms of Harish-Chandra modules. We will show how these categories of Harish-Chandra modules behave with respect to the existence of "big" projectives, double centralizer properties and tilting modules.

On infinte dimensional Lie algebras.

Manuel Forero Piulestán
University of Cádiz, Spain
ForeroManuel@hotmail.com
Coauthors: Antonio Jesús Calderón Martín.

We introduce the concept of Cartan decomposition relative to a Cartan subalgebra H, in sense [1], for Lie algebras of arbitrary dimension. The class of simple Lie algebras having such decomposition is a wide class of Lie algebras closely related to the topologically simple L*-algebras, the simple c-involutive Lie algebras and the topologically simple compact Banach-Lie algebras. We give a description theorem for the complex ones, turning out to be the natural extension of the finite dimensional setup.

References

[1] Y. Billig and A. Pianzola, On Cartan subalgebras, J. Algebra, 171 (1995), 397–412.

The s-tame dimension vectors for stars

Angela Holtmann
Universität Bielefeld, Germany
aholtman@mathematik.uni-bielefeld.de

A quiver $Q = (Q_0, Q_1)$ is given by a set of vertices Q_0 and a set of arrows Q_1 and two maps $s, t : Q_1 \to Q_0$ which assign to every arrow $\alpha \in Q_1$ its starting point $s(\alpha)$ and its terminating point $t(\alpha)$. A star Q is a quiver without multiple arrows and without oriented cycles, with a unique sink which is the only possible branching vertex. (Thus Q is obtained from linearly oriented quivers of type $\mathbb{A}$ by identifying the sinks.)

A subspace representation of a star $Q = (Q_0, Q_1)$ is a collection of vector spaces V_i, $i \in Q_0$, over a particular field K together with K-linear maps $V_\alpha : V_{s(\alpha)} \to V_{t(\alpha)}$, $\alpha \in Q_1$, which are all injective.

In 1999, P. Magyar, J. Weyman, and A. Zelevinsky classified all dimension vectors for stars, which occur for only finitely many isomorphism classes of subspace representations (see [MWZ]). Since a subspace representation is a vector space with multiple (partial) flags, and isomorphism classes of subspace representations with an n-dimensional vector space V at the central point are in 1-1 correspondence with the orbits of GL_n under conjugation in the multiple flag variety of V, this is the classification of the multiple flag varieties of finite type.

A "continuation" of this is the classification of all "s-tame" dimension vectors of stars (see [H]). A dimension vector $\mathbf{d}$ is called s-tame, if there exists at least one one-parameter family of subspace representations, but for every decomposition of $\mathbf{d}$ as a sum of dimension vectors of subspace representations there is no n-parameter family of subspace representations with $n \geq 2$ for either of the summands. In this way, the problem of multiple flag varieties of tame type is solved.

I would like to show how one can find the s-tame dimension vectors (and create a complete list) and to give a characterisation of the s-tame dimension vectors in terms of the Tits forms corresponding to the stars.

For the latter part one can use a theorem by V.G. Kac which he has proven in [K]:

Let K be an algebraically closed field. There is an indecomposable representation over K for a dimension vector $\mathbf{d}$ of a quiver Q, if and only if $\mathbf{d}$ is a positive root of the corresponding symmetric Kac-Moody Lie algebra $\mathfrak{g}(Q)$. Furthermore, in case $\mathbf{d}$ is a positive root, the maximal number of parameters, on which a family of representations with dimension vector $\mathbf{d}$ depends, is given by $1 - q(\mathbf{d})$, where q denotes the Tits form corresponding to the quiver Q.

References

[H] A. Holtmann, The s-tame dimension vectors for stars (Dissertation, Fakultät für Mathematik,Universität Bielefeld) (2003);
available at http://archiv.ub.uni-bielefeld.de/disshabi/2003/0029/

[K] V.G. Kac, Root systems, Representations of Quivers and Invariant Theory, Lecture Notes in Mathematics, Invariant Theory, pp. 74 – 108 (1982)

[MWZ] P. Magyar, J. Weyman, A. Zelevinsky, Multiple Flag Varieties of Finite Type, Advances in Mathematics 141, pp. 97–118 (1999)

Representations of a quiver with automorphism

Andrew Hubery
Fakultät für Mathematik, Universität Bielefeld, Germany
`hubery@mathematik.uni-bielefeld.de`

In [Kac], Kac proved that the dimension vectors of indecomposable representations of a quiver are precisely the positive roots of the associated symmetric Kac-Moody Lie algebra. Moreover, he showed that over a finite field, the number of isomorphism classes of absolutely indecomposable representations of a given dimension vector is given by a polynomial in the size of the field. He then conjectures that the constant term of this polynomial equals the multiplicity of the root in the Lie algebra.

We have generalised these ideas to include all symmetrisable Kac-Moody Lie algebras, by considering a quiver with an admissible quiver automorphism (c.f. [Lus]). We consider the subcategory with objects those representations isomorphic to their 'twist' by the automoprhism, the so-called isomorphically invariant (or ii-) representations.

We show that the dimension vectors of the ii-indecomposables are precisely the positive roots of the associated symmetrisable Kac-Moody Lie algebra, and that over a finite field, the number of isomorphism classes of absolutely ii-indecomposables is polynomial in the size of the field.

By analysing these polynomials for the affine quivers, we conjecture that the constant terms are given by the weight multiplicities of a naturally occurring integrable module for the symmetrisable Lie algebra - namely the fixed point subalgebra of a quiver under a diagram automorphism.

References

[Hub] A. Hubery, 'Quiver representations respecting a quiver automorphism: a generalisation of a theorem of Kac', preprint (math.RT/0203195).

[Kac] V.G. Kac, 'Infinite root systems, representations of graphs and invariant theory', Invent. Math. 56 (1980), 57–92.

[Lus] G. Lusztig, 'Introduction to quantum groups' Progress in Mathematics 110 (Birkhäuser, 1993).

The derived length of Lie soluble group algebras

Tibor Juhász

University of Debrecen, Hungary

juhaszti@math.klte.hu

In Shalev's paper was given an upper bound on derived length of Lie soluble group algebra KG. We investigate the question when this bound is exact on 2-groups. Furthermore we determine the derived length of group algebras for some 2-groups.

Abelian categories, almost split sequences, and comodules

Mark Kleiner
Syracuse University, USA
mkleiner@syr.edu
Coauthors: Idun Reiten

The following are equivalent for a skeletally small abelian Hom-finite category over a field with enough injectives and each simple object being an epimorphic image of a projective object of finite length. (a) Each indecomposable injective has a non-zero socle. (b) The category is equivalent to the category of socle-finitely copresented right comodules over a right semiperfect and right cocoherent coalgebra such that each simple right comodule is socle-finitely copresented. (c) The category has left almost split sequences.

Zeros of Coxeter and reciprocal polynomials

Piroska Lakatos
University of Debrecen, Intitute of Mathematics and Informatics, Hungary
lapi@math.klte.hu
Coauthors: L. Losonczi

We study the spectral properties of the Coxeter transformation of some oriented graphs without oriented cycles.

As Coxeter polynomials are reciprocal, the location of zeros of such polynomials is an important issue. We give sufficient conditions for reciprocal polynomials to have all their zeros on the unit circle.

References

[1] Boldt, A., Takane, M., *The spectral classes of unicyclic graphs*, Journal of Pure and Applied Algebra **133** (1998), 39–49.

[2] Lakatos, P., *Zeros of reciprocal polynomials*, Publ. Math. **61/3-4** (2002), 645–661.

[3] Ringel, C. M., *The spectral radius of Coxeter transformations for generalized Cartan matrix*, Math. Ann. **300** (1994), 331–339.

Locally hereditary tame algebras

Zbigniew Leszczynski
Nicolaus Copernicus University, Torun, Poland
zbles@mat.uni.torun.pl

A finite dimensional algebra A over a field K is called locally hereditary if each local submodule of any projective A-module is projective. The class of locally hereditary algebras contains the hereditary algebras and the incidence algebras of partially ordered posets.

We give the several equivalent characterizations of the tame locally herditary algebras over an algebraically closed field K.

On Injectivity of Projection Algebras

Mojgan Mahmoudi
Shahid Beheshti University, Tehran, Iran.
m-mahmoudi@cc.sbu.ac.ir
Coauthors: M. Mehdi Ebrahimi

A projection algebra is an M-sets for the monoid $M = (\mathbb{N}^\infty, min)$. Projection algebras were first introduced by Computer scientists as an algebraic version of ultrametric spaces. They use this notion as a convenient mean for algebraic specification of process algebras.

Some kinds of injectivity of a class of projection algebras, such as s-injectivity of separated projection algebras, have been already studied. Here, we introduce m and p-injectivity of projection algebras and show, among other things, that injectivity, s-injectivity, and m-injectivity coincide.

Also, in contrast to the case of modules it is well-known that Baer Criterion does not generally hold for injectivity of M-sets for arbitrary monoid M. Here, we prove that Baer Criterion does hold for injectivity of projection algebras.

Schur superalgebras in characteristic p

Frantisek Marko
Pennsylvania State University Hazleton, USA
fxm13@psu.edu
Coauthors: Alexandr N. Zubkov

It is well known that Schur algebras associated with semisimple algebraic groups for any characteristic are quasi-hereditary and cellular. Muir introduced Schur superalgebras and showed that they are semisimple in the characteristic zero case and also when the characteristic of the ground field is sufficiently large. We show that in the case of a "small" characteristic, there are Schur superalgebras which are neither quasi-hereditary, nor cellular, nor stratified.

Tilting modules and cluster algebras
Robert Marsh
University of Leicester, England
rjm25@mcs.le.ac.uk
Coauthors: Markus Reineke and Andrei Zelevinsky

Cluster algebras were recently defined by S. Fomin and A. Zelevinsky in an attempt to describe the Lusztig-Kashiwara canonical basis of a quantum group, and to develop an algebraic framework for total positivity in semisimple algebraic groups. In this work they associate to each finite root system a smooth complete simplicial fan which, according to work by F. Chapoton, S. Fomin and A. Zelevinsky, is the normal fan of a polytope which can be regarded as a generalised version of the associahedron, or Stasheff polytope (in type A, it is this polytope).

We provide a quiver-theoretic interpretation of these fans; their main properties then become easy consequences of the known facts about tilting modules over the path algebras of Dynkin quivers due to K. Bongartz, D. Happel and C. M. Ringel.

Splitting properties for rational modules
Constantin Nastasescu
University of Bucharest, Romania
cnastase@al.math.unibuc.ro
Coauthors: Blas Torrecillas

Let C be a coalgebra and C^* the dual algebra associated to C. We say that C has the splitting property if for any rational left C^*-module, the rational part of M, $Rat(M)$, is a direct summand of M. We will show that every coalgebra C with splitting property is finite dimensional.

Radicals and socles of an algebra without identity
M. Teresa Nogueira
Universidade de Lisboa, Portugal
tnogueir@cii.fc.ul.pt
Coauthors: M. Luísa Galvão, J. A. Green

In 1945, N. Jacobson has introduced the concept of radical of a ring A (which is known as "Jacobson radical", and is denoted $J = J(A)$). Later the notion of (Jacobson) radical of a left (or right) A-module M, $J(M)$, has been defined as the intersection of all submodules N of M such that M/N is simple. Thus one may consider the left radical $Jl = J({}_AA)$ of A considered as a left module over itself and the right radical $Jr = J(A_A)$ of A considered as a right module over itself. These are bilateral ideals of A, and are contained in $J(A)$. If A

has identity, one has $J = Jl = Jr$, but this equality is not valid in general. Dually, it is possible to define left socle Sl and the right socle Sr of A. We shall establish relations between J, Jl, Jr, Sl and Sr, and for artinian algebras we shall obtain expressions for $Jl(A)$, $Jr(A)$, $Sl(A)$ and $Sr(A)$. In particular, if A is a finite dimensional algebra over a field we display $Jl(A)$ (and $Jr(A)$) in a matrix representation.

P.I. algebras with involution

Tsetska Rashkova
University of Rousse "Angel Kanchev", Bulgaria
`tcetcka@ami.ru.acad.bg`

The talk is a survey on results concerning matrix algebras with symplectic involution and identities for them in symmetric or skew-symmetric variables. The main results are connected with:
- the minimal degree of the identities in symmetric variables;
- the description of Bergman type identities in any of the two types of variables;
- structure results on Bergman type identities;
- the minimal degree of Bergman type identities for lower order matrix algebras.
The talk ends with the possible generalizations of the given results in the case of superalgebras and superinvolution.

Generalization of function ρ and representations of quivers in Hilbert spaces

Igor K. Redchouk
Institute of Mathematics, Kiev, Ukraine
`red@imath.kiev.ua`
Coauthors: A.V. Roiter

A numeric function ρ:

$$\rho(k) = 1 + (k-1)/(k+1), k \in N$$

was considered in [1]. In its terms criterions of finite representability and tameness of marked quivers, posets with equivalence and dyadic posets can be obtained; Dynkin schemes and extended schemes also can be characterized.

Define function ρ on set $V = \{(x_1, ...x_s) | x_i \in N;\ x_i \geq x_j \text{ if } i < j;\ i,j = 1, ...s\}$ in such a way: $\rho(x_1, ...x_s) = \sum\limits_{i=1}^{s} \rho(x_i)$. Define then a family of functions

$$\rho_n(k) = 1 + (u_{k-1})/(u_{k+1})$$

where u_k is a sequence of natural numbers defined recurrently: $u_0 = 0$, $u_1 = 1$, $u_{i+2} = nu_{i+1} - u_i$. Hence $\rho = \rho_2$. The function ρ_n is defined on V analogously to ρ.

It is considered an equation

$$\rho_n(x_1, ...x_s) = n + 2 \ (1)$$

which has important employments when $t = 2$ [1]. It turned out that when $t > 2$ solutions of (1) are obtained from solutions of case $t = 2$ (there are only four of them: $(1,1,1,1)$, $(2,2,2)$, $(3,3,1)$, $(5,2,1)$) by adding $t-2$ numbers equal to maximal coordinate. It is also shown that the equation (1) has no solutions when $t \in Q^+ \setminus N$.

It is considered a relation of functions ρ_n with representations of quivers over field and in categories of Hilbert spaces [2,3,4], and respective Coxeter functors. A case $t = 2$ corresponds to extended Dynkin schemes.

References

[1] L.A. Nazarova, A.V. Roiter. *Norm of a relation, separating functions and representations of marked quivers.* Ukr. Math. Jour., 54(2002), No.6, p.808-840.

[2] I.N. Bernshtein, I.M. Gelfand, V.A. Ponomariov. *Coxeter functors and Gabriel's theorem.* Izv. Mat. Nauk, 28(1973), p.19-33.

[3] S.A. Kruglyak, V.I. Rabanovich, Yu.S. Samoilenko. *On sums of projectors.* Func. An. and App., 34(2002), No.3, p.30-35.

[4] S.A. Kruglyak, A.V. Roiter. *Locally-scalar representations of graphs in the category of Hilbert spaces.* Prepr. Ukr. Math. Jour. (2003).

Structure and representations of conformal algebras

Alexander Retakh
Massachusetts Institute of Technology, USA
`retakh@math.mit.edu`

Conformal algebras were introduced originally to describe the singular part of the OPE formula in the theory of vertex algebras; they are also related to Hamiltonians in the formal calculus of variations.

In this talk I will mostly focus on conformal algebras $Cend_n$ and their Lie and associative subalgebras (their role is analogous to that of matrix algebras in "ordinary" algebra). The Lie subalgebras, in particular, correspond to a class of Lie algebras of quadratic growth with an interesting representation theory. I will discuss known results and state several conjectures.

Finite dimensional representations of invariant differential operators

Sonia L. Rueda
Universidad de Almería, Spain
srueda@ual.es
Coauthors: Ian M. Musson

Let k be an algebraically closed field of characteristic 0, $Y = k^r \times (k^\times)^s$ and let G be an algebraic torus acting diagonally on the ring of differential operators $\mathcal{D}(Y)^G$. We give necessary and sufficient conditions for $\mathcal{D}(Y)^G$ to have enough simple finite dimensional representations, in the sense that the intersection of the kernels of all the simple finite dimensional representations is zero. As an application we show that if $K \longrightarrow GL(V)$ is a representation of a reductive group K and if zero is not a weight of a maximal torus of K on V, then $\mathcal{D}(V)^K$ has enough finite dimensional representations. We also construct examples of FCR-algebras with any GK dimension ≥ 3.

Some results on the classification of quasiassociative algebras

Ana Paula Santana
Universidade de Coimbra, Portugal
aps@mat.uc.pt
Coauthors: Helena Albuquerque

Given the finite group G and the field K, let $A = \oplus_{g\in G} A_g$ be a G-graded K-algebra. We say that A is a quasiassociative algebra if, for homogeneous elements $x \in A_g, y \in A_h, z \in A_l$, we have $(xy)z = \phi(g,h,l)x(yz)$, where $\phi : G \times G \times G \to K \setminus \{0\}$ is a cocycle in G. We study classes of quasiassociative algebras with "well behaved" null part, generalizing well known results for associative algebras.

Homologically induced modules for Lie superalgebras

José Carlos Santos
Universidade do Porto, Portugal
jcsantos@fc.up.pt

A general description will be made of the method of (co)homological induction for Lie superalgebras. It will be seen that the study of these modules is much more complex in the case of basic classical Lie superalgebras than in the case of simple Lie algebras. For instance, it will be proved that not every finite dimensional irreducible module can be obtained by (co)homological induction starting form a fixed Borel subalgebra and a Verma module.

On Galois coverings of selfinjective algebras

Andrzej Skowronski
Nicolaus Copernicus University, Torun, Poland
skowron@mat.uni.torun.pl
Coauthors: Kunio Yamagata

Frequently, selfinjective algebras are Morita equivalent to selfinjective algebras which are socle deformations of selfinjective algebras having triangular Galois coverings, and then we may reduce the study of such algebras and their representations to that for the corresponding algebras of finite global dimension. This is the case for all representation-finite selfinjective algebras and important classes of tame selfinjective algebras (selfinjective algebras of polynomial growth, tame enveloping algebras of restricted Lie algebras, tame Hopf algebras of infinitesimal groups) over algebraically closed fields.

We give necessary and sufficient conditions for a basic connected selfinjective artin algebra A to admit a Galois covering by the repetitive algebra of a factor algebra B of A, with infinite cyclic Galois group.

Does the Murphy subalgebra of the Hecke algebra have a Grobner-Shirshov basis?

Adrian Williams
Cornwall, UK
alwleon@hotmail.com

Grobner-Shirshov basis theory for associative algebras and their representations has been used by S.-J.Kang et al (J.Alg. 252 (2002), 258-292) to construct Specht modules in terms of generators and relations, and to obtain a monomial basis parametrized by certain semi-standard Young tableaux. The subalgebra of the Iwahori-Hecke algebra for the symmetric group generated by the Murphy operators has dimension equal to the number of standard Young tableaux. It is conjectured that over a field of characteristic p the subalgebra has a monomial basis. It is an open question whether the subalgebra is maximal commutative.

On local embeddability of groups and group algebras

Milos Ziman
Comenius University, Bratislava, Slovakia
ziman@fmph.uniba.sk

Let Q be a grupoid and $\mathcal{F}$ be some class of grupoids. Then Q is said to be *locally embeddable into the class* $\mathcal{F}$ if for any finite subset M of Q there is an $F \in \mathcal{F}$ and an injective map $\varphi : (MUM^2) \to F$ such that $\varphi(xy) = \varphi(x)\varphi(y)$ for every $x, y \in M$.

Let A be a algebra with involution and $\mathcal{H}$ be some class of not necessarily associative algebras. Then A is said to be *locally embeddable into the class* $\mathcal{H}$ if for any finite subset M of A there is an $H \in \mathcal{H}$ and an injective linear map $f : span(MUM^2) \to H$ such that for every $u, v \in M$ we have $f(uv) = f(u)f(v)$.

Every group is localy embeddabel into the class of finite loops (quasigroups with the unit) with inverses satisfying the identity $(xy)^{-1} = y^{-1}x^{-1}$ (IAA loops for short).

Hence every group algebra with natural involution (arising from inverse map of the group) is locally embedable into the class of loop algebras (with natural involution) of finite IAA loops. (There is an aditional condition to the map f - preserving of involution.)

The local embeddability of group G into the class of finite groups is equivalent to the local embeddability of the group algebra of G into the class of finite dimensional (associative) algebras.

On the zero set of semi-invariants for quivers

Grzegorz Zwara
Nicolaus Copernicus University, Torun, Poland
gzwara@mat.uni.torun.pl
Coauthors: Christine Riedtmann

Let d be a prehomogeneous dimension vector for a finite quiver Q. We show that the common zeroes of all semi-invariants of positive degree for the variety of representations of Q with dimension vector $N * d$ under the product of the general linear groups at all vertices is irreducible and a complete intersection for large natural numbers N.

Module Theory

Complexity of degenerations of modules
Klaus Robert Aehle
ETH Zuerich, Switzerland
aehle@math.ethz.ch
Coauthors: Christine Riedtmann, Grzegorz Zwara

A module M over an associative algebra A over an algebraically closed field k is said to degenerate to a module N if N belongs to the closure of the isomorphism class of M in the algebraic variety of d-dimensional modules, d a natural number. By studying a class of $A - k[[t]]$ bimodules related to a degeneration of modules we associate a non-negative integer to a degeneration of modules, its complexity, and study its properties.

Comatrix Corings
Khaled Al-Takhman
Birzeit University, Palestine
takhman@birzeit.edu

Starting from a finitely generated projective generator module we give a construction of comatrix corings generalizing the notion of comatrix coalgebras. We also establish an equivalence (Morita-Takeuchi equivalence) a coring and its comatrix coring. It is also shown that the comatrix coring of a coring is the coendomorphism coring of a certain comodule.

Tilting modules and Gorenstein rings
Lidia Angeleri Hügel
Università degli Studi dell' Insubria, Varese, Italy
lidia.angeleri@uninsubria.it
Coauthors: Dolors Herbera, Jan Trlifaj

A two-sided noetherian ring R with the injective dimensions of $_RR$ and R_R both being finite is called a (Iwanaga-)Gorenstein ring.

We give some new characterizations of such rings in terms of tilting theory. As a consequence, we show that the first finitistic dimension conjecture holds true for every Gorenstein ring. Moreover, we prove that Gorenstein injectivity, and flatness, can be tested by finitely generated modules of finite projective dimension, improving a recent result by Enochs and Jenda.

Spectral Torsion Theories and a General Theory of Types

María José Arroyo Paniagua
Universidad Autónoma Metropolitana-Iztapalapa, Mexico
mja@xanum.uam.mx
Coauthors: José Ríos Montes

It is a well known fact that, for an associative ring with unit and for a given spectral torsion theory, the ring of quotients is regular selfinjective. Also, the lattice of generalization of the spectral torsion theory is isomorphic to the lattice of central idempotents of the ring of quotients. Using a General Theory of Types, which generalizes the Kaplansky Theory of Types, we introduce a decomposition of the lattice of generalization of the spectral torsion theory given. With these decomposition, we give a new stucture theorem for the ring of quotients.

On divisible modules with respect to a torsion theory

Septimiu Crivei
Babes-Bolyai University of Cluj-Napoca, Romania
crivei@math.ubbcluj.ro
Coauthors: Iuliu Crivei

For a hereditary torsion theory T, a module is called T-divisible if it is injective with respect to every monomorphism having a T-torsionfree cokernel. We especially study self-T-divisible modules, which may be seen as generalizations of the recently reconsidered T-complemented modules.

Torsion free covers of cotorsion modules

Sergio Estrada
University of Almeria, Spain
sestrada@ual.es

In this talk we elaborate on a result of J.Xu and show that in some situations there is close relations between various Galois and coGalois groups. Then we use this to extend known results about the coGalois group (as topological group) of torsion free covers.

Projective modules and divisor homomorphisms

Alberto Facchini
Universitá di Padova, Italy; Centre de Recerca Matematica, Barcelona, Spain
facchini@math.unipd.it
Coauthors: Franz Halter-Koch

For a ring R, let V(R) denote the monoid whose elements are all isomorphism classes of finitely generated right R-modules, and whose operation is the operation induced by direct sum. We shall present some results relating the ring structure of the ring R and the monoid structure of the reduced commutative monoid V(R).

Triangulated categories and the Ziegler spectrum

Grigory Garkusha
Int. Centre Theoretical Physics, Trieste, Italy
garkusha@ictp.trieste.it
Coauthors: Mike Prest

The Ziegler spectrum of a ring introduced by M.Ziegler in model-theoretic terms was intensively being studied in the 90-s (W.Crawley-Boevey, I.Herzog, H.Krause, M.Prest). In a recent paper (2002), H.Krause has defined the Ziegler topology for compactly generated triangulated categories. In our talk we shall discuss the realtionship between the Ziegler spectrum of a ring with that of related triangulated categories.

Cotilting dualities

Enrico Gregorio
Università di Verona, Italy
gregorio@sci.univr.it

It is possible to extend the theory of cotilting dualities to suitable subcategories of modules over generalized Artin algebras; the theory is partly parallel to the classical one for Artin algebras, but presents significant differences. An application is the study of cotilting dualities for comodules over coalgebras.

Weakly Noetherian or Artinian modules and rings

El-Amin Kaidi
Universidad de Almeria, Spain
`elamin@ual.es`
Coauthors: Faouzi Latifa and Hmaimou Abdelilah

An R-module M is called weakly noetherian (resp. artinian) if for every endomorphism f of M the chain $Ker(f) \subset Ker(f^2) \subset ...$ (resp. $Im(f) \supset Im(f^2) \supset ...$) is finite. The class of weakly noetherian (resp. artinian) modules lies properly between the class of noetherian (resp. artinian) and the class of Hopfian (resp. co-Hopfian) modules. Some properties of weakly noetherian (resp. artinian) modules and rings are obtained. A version of the Hopkins-Levitzki (resp. Kaplansky) Theorem for artinian rings (resp. Banach algebras) is proved. Namely, a weakly artinian ring (resp. Banach algebra) is weakly noetherian (resp.algebraic).

The shape of indecomposable injective modules over non-commutative noetherian rings

Thomas G. Kucera
University of Manitoba, USA
`tkucera@cc.umanitoba.ca`

Since the seminal paper of E. Matlis (1958) the structure of the indecomposable injectives over a commutative noetherian ring R has been very well understood.Over a non-commutative noetherian ring none of Matlis-style structure works in complete generality. There is also a wide range of apparently pathological examples, pointing towards the difficulty of making any general statements about the structure of the indecomposable injectives.

However, in first-order logic, the complete theory of an indecomposable injective (right) module over a (right) noetherian ring falls into one of the simplest and most well behaved model-theoretic classifications. In any such module we can define the elementary socle series: a first-order definable analogue of the ordinary socle series. This series is an ascending sequence of definably closed submodules whose union exhausts the module, but we know nothing in general about its structure beyond that one simple fact.

In this talk I will discuss two examples, both held to be ones where the structure of the indecomposables should be particularly intractable. In my recent paper [1] I gave a detailed description of the indecomposable injectives over some pathological right noetherian rings of Jategaonkar. My PhD student Alina Duca is studying the indecomposable injective modules over Weyl algebras. In both cases, the ordinary socle series fails to give a useful description of the indecomposables, but the elementary socle series yields a nice description.

References

[1] T. G. Kucera, Explicit descriptions of the indecomposable injective modules over Jategaonkar's rings. Comm. Alg 30(12) 6023–6054, 2002.

Weil modules

Miroslav Kures
Brno University of Technology, Czech Republic
kures@mat.fme.vutbr.cz

For a local (Weil) R-algebra A, let V be A-module, which is finite-dimensional as a real vector space. We study such modules and present new results having applications in differential geometry, especially for the geometry of induced product preserving bundle functors $T^{A,V}$.

Central closure for ring extensions with additional module structure

Christian Lomp
Universidade do Porto, Portugal
clomp@fc.up.pt

Let $A \subseteq B$ be a ring extension of associative rings (with unit). We say that this ring extension has an additional module structure if A is a left B-module such that the restriction of the B-action to A is equal to the A-action on A itself. There are many examples of those ring extensions, e.g. $B := A \otimes A^{op}$ or $B = A\#G$ the skewgroup ring of A and a group G that acts as automorphisms on A or more generally $B = A\#H$ where A is a left H-module algebra. In general one might think of B as some subalgebra of $End(A)$ which contains A.

Assuming that B is such a subalgebra of $End(A)$ which contains the multiplication algebra $M(A)$ of A and assuming that A has no B-stable nilpotent ideals, i.e. A is B-semiprime, we give a general construction of a central closure of A which mimics the ordinary central closure of a semiprime algebra. This construction follows Wisbauer's approach defining the central closure of a semiprime algebra using $M(A)$. More precisely we show that A is a polyform B-module and that the self-injective hull of A as B-module carries a canonical ring structure extending the one of A.

Finally we will apply our construction to Hopf actions on algebras and compare it with the central closure of module algebras introduced by J.Matczuk.

On the existence of finite Galois stable groups

Dmitry Malinin
Belarusian State Pedag. Univ., Belarus
malinin@bspu.unibel.by

Let E be a finite extension of a number field F with Galois group Γ, and let O_E and O_F be the maximal orders of E and F. Let O'_E be the intersection of valuation rings of all ramified prime ideals in the ring O_E, and let $O'_F = F \cap O'_E$. Denote $\phi_E(t) = [E(\zeta_t) : E]$ where ζ_t is a primitive t-root of 1. Let $F(G)$ be a field obtained via adjoining to F all matrix coefficients of all matrices $g \in G \subset GL_n(E)$.

Theorem 1. *1) For a given number field F and integers n and t, there is only a finite number of normal extensions E/F such that $E = F(G)$ and G is a finite abelian Γ-stable subgroup of $GL_n(O_E)$ of exponent t.*

2) For a given number field F and integers n and $d = [E\colon F]$, there is only a finite number of fields $E = F(G)$ for some finite Γ-stable subgroup G of $GL_n(O_E)$.

Theorem 2. *Let $d > 1, t > 1$ and $n \geq \phi_E(t)d$ be given integers, and let E/F be a given extension of degree d. Then there is an abelian Γ- stable subgroup $G \subset GL_n(E)$ of exponent t such that $E = F(G)$.*

Theorem 3. *Let $d > 1, t > 1$ be given rational integers, and let E/F be an unramified extension of degree d.*
1)If $n \geq \phi_E(t)d$, there is a finite abelian Γ- stable subgroup $G \subset GL_n(O'_E)$ of exponent t such that $E = F(G)$.
2)If $n \geq \phi_E(t)dh$ and h is the exponent of the class group of F, there is a finite abelian Γ-stable subgroup $G \subset GL_n(O_E)$ of exponent t such that $E = F(G)$.
3)If $n \geq \phi_E(t)d$ and h is relatively prime to n, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.
4)If d is odd, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.

In all cases above G can be constructed as a group generated by matrices $g^\gamma, \gamma \in \Gamma$ for some $g \in GL_n(E)$.

Theorem 4. *Let E/F be a given extension of degree d, and let $G \subset GL_n(E)$ be a finite abelian Γ-stable subgroup of exponent t such that $E = F(G)$ and n is the minimum possible. Then $n = d\phi_E(t)$ and G is irreducible under conjugation in $GL_n(F)$. Moreover, if G has the minimal possible order, then G is a group of type $(t, t, ..., t)$ and order t^m for some integer $m \leq d$.*

Theorem 5. Let K/Q be a normal extension with Galois group Γ, and let $G \subset GL_n(O_K)$ be a finite Γ-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where K_{ab} is the maximal abelian over Q subfield of K.

Topologies and functors between categories of modules for nonunital rings

Leandro Marin
Departamento de Matematica Aplicada, Universidad de Murcia, Spain
leandro@um.es

Let X be a set with the discrete topology and $X^{\mathbb{N}}$ be topological product of copies of X. This is metric 0-dimensional topological space. Compact subspaces of $X^{\mathbb{N}}$ play a central role in the study of the structure of the category of firm modules $R - \mathsf{DMod}$ for a nonunital ring R. Firm modules are those M such that the canonical homomorphism $R \otimes_R M \to M$ with $r \otimes m \mapsto rm$ is an isomorphism. We will show those relations and also how the construction of functors $F : R - \mathsf{DMod} \to R' - \mathsf{DMod}$ that preserve colimits are connected with the continuous extensions of functions over the topological space $X^{\mathbb{N}}$.

Semiprime preradicals and semiprime modules.

Federico Raggi
Universidad Nacional Autònoma de Mèxico, Mèxico
fraggi@matem.unam.mx
Coauthors: Rogelio Fernàndez-Alonso, Hugo Rincòn, Josè Rìos, Carlos Signoret

We give definitions of semiprime preradicals and semiprime modules, and we study some relations between these concepts, using the lattice structure of the class of all preradicals developed in previous papers. Finally we give some characterizations of rings that have certain conditions on semiprime radicals.

Generalized E-rings

Lutz Struengmann
University of Duisburg-Essen, Germany
lutz.struengmann@uni-essen.de
Coauthors: Ruediger Goebel, Saharon Shelah

A ring R is called an E-ring if the canonical homomorphism from R to the endomorphism ring $End(R^+)$ of the additive group R^+, taking any $r \in R$ to the endomorphism left multiplication by r is an isomorphism of rings. In this case R^+ is called an E-group. Subrings of the rationals are obvious examples of E-rings. However there is a proper class of examples constructed recently. E-rings come up naturally in various topics of algebra, so it's not surprising that they were investigated thoroughly in the last decade. This also led to a generalization: an abelian group G is an EE-group if there is an epimorphism from G onto the additive group of $End(G)$. If G is torsion-free of finite rank, then G is an E-group if and only if it is an EE-group. The obvious question whether the classes of E-groups and EE-groups coincide in general was raised a few years ago. We will answer it by showing that the two notions differ in the infinite rank case. Applying combinatorial machinery to non-commutative rings we shall produce an abelian group G with (non-commutative) $End(G)$ and the desired epimorphism with prescribed kernel H. Hence, if we let $H = 0$, we obtain a non-commutative ring R such that $End(R^+) \cong R$ but R is not an E-ring.

Applying Torsion Theories to Finite von Neumann algebras

Lia Vas
University of the Sciences in Philadelphia, USA
l.vas@usip.edu

Von Neumann algebras facilitate the study of certain topological spaces. The author uses torsion theories to derive a series of results that expand the understanding of modules over a finite von Neumann algebra A. The presentation focuses on some torsion theories for A and how they can be used to obtain new results on the ring theoretic properties of the algebra A or improve existing results on L^2-invariants of A-modules.

Coprime Coalgebras and Dual Algebras

Indah Emilia Wijayanti
Gadjah Mada Univ., Jogjakarta, Indonesia
indah@math2.uni-duesseldorf.de

For any coalgebra C over field k we can consider its dual algebra C* = Hom(C,k). It is already defined a coprime subcoalgebra through wedge product. In this paper we define further a coprime coalgebra and yield the following equivalence, C is coprime if and only if C* is prime.

Non-commutative Algebraic Geometry

Structure of null sets in the plane and some applications of geometric measure theory

Samuel Babatunde Akinremi
Gambia College, Serekunda, The Gambia
samuel_akinremi@yahoo.com

The starting point is a decomposition theorem for null sets (sets of measure zero) in the plane which can be obtained as a direct consequence of a geometric version of the Erdos-Szekeres theorem. I will outline some applications to problems in Goemetric Measure Theory (and not only, and if time permits, I will address the issue of extending these results to higher dimension).

Set-theoretic Solutions of the Yang-Baxter Equations

Tatiana Gateva-Ivanova
University of Antwerp, Belgium and Institute of Mathematics, Bulgarian Academy of Sciences, Bulgaria
tatianagateva@yahoo.com

A bijective map $r : X^2 \longrightarrow X^2$, where X is a finite set, is called a *set-theoretic solution of the Yang-Baxter equation* (YBE) if the braid relation $r_{12}r_{23}r_{12} = r_{23}r_{12}r_{23}$ holds in X^3.

The close relations between the set-theoretic solutions of YBE, the semigroups of I-type, and the skew-polynomial rings with binomial relations were studied in a joint paper with Michel Van den Bergh.

In this talk I will discuss the class of nondegenerate involutive square-free solutions $(X;r)$ and the properties of the related "Yang-Baxter" algebraic structures- the semigroup $S(r)$, the group $G(r)$, and the semigroup algebra $kS(r)$ over a field k. (Each of these structure is generated by X and has the same set of quadratic defining relations $R(r)$ naturally determined by r.) In particular, I will discuss how are these properties related to a conjecture of mine that for any solution (X,r) the set X can be ordered so that the algebra $A = kS(r)$ has a PBW basis. This implies a later conjecture of Etingof-Schedler that each solution $(X;r)$ can be presented as an extension of two disjoint "smaller solutions" $(Y;r_Y)$ and $(Z;r_Z)$. Furthermore I use the nice combinatorial properties of $(X;r)$ to describe the set of extensions $Ext(Y,Z)$, where $(Y;r_Y)$ and $(Z;r_Z)$ are arbitrary disjoint solutions. In particular an explicit description of the automorphism group $Aut(X;r)$ is used to classify the so called *left extensions* of $Ext_+(X,Y)$.

Projective coordinate algebras of exceptional curves

Dirk Kussin
University of Paderborn, Germany
dirk@math.upb.de

We study the geometry of exceptional curves (in the sense of Lenzing) over arbitrary fields. Exceptional curves are the parameter curves for separating tubular families for tame hereditary algebras and for canonical algebras (in the sense of Crawley-Boevey and Ringel). Moreover, the module category over such a finite dimensional algebra is derived-equivalent to the category of coherent sheaves over the corresponding exceptional curve. Therefore, the geometry and the representation theory is strongly related. Our aim is to find and describe projective coordinate algebras for exceptional curves with "nice" ring theoretical properties in order to get a more explicit approach to the geometry. In general, these coordinate rings will be non-commutative. Actually, they are commutative only in very special cases.

For simplicity, we restrict our presentation to the case, where the exceptional curve $\mathbb{X}$ is *homogeneous*, that is, $\mathbb{X}$ is the parameter curve for a tame bimodule algebra (which were studied by Dlab and Ringel). This is an example for a non-commutative projective scheme in the sense of Artin-Zhang.

Let k be an arbitrary field and $\mathbb{X}$ be a homogeneous exceptional curve over k. Let $\mathcal{H} = \text{coh}(\mathbb{X})$ be the associated category of coherent sheaves. Our main result is, that for *each* such curve there is an automorphism s of $\mathcal{H}$ and a line bundle L such that the pair (L, s) is ample, hence the orbit algebra R, which is the direct sum of all $\text{Hom}(L, s^n L)$ $(n \in \mathbb{Z})$, is a projective coordinate algebra of $\mathbb{X}$, that is, $\mathcal{H}$ is the quotient category of $\text{mod}^{gr}(R)$ modulo the objects of finite length, and we have the *additional property*, that each homogeneous prime ideal of height one in R is principal, generated by a normal homogeneous element. We give an explicit description of the correspondence between primes and points.

As special cases of these "factorial" orbit algebras we get the (small) preprojective algebras defined by Baer, Geigle and Lenzing, and hence also get new results for this class of algebras.

We discuss further properties of our orbit algebras (like commutativity, module-finiteness over the center, unique factorization) and present explicit examples.

Deformations of abelian categories

Wendy Tor Lowen
Free University of Brussels, Belgium
wlowen@vub.ac.be
Coauthors: Michel Van den Bergh

We introduce a notion of deformations of abelian categories, which generalizes what happens to module categories under a classical deformation of algebra's. Under a suitable flatness condition, certain categorical/homological properties lift under deformation and we obtain several equivalences of deformation problems.

Ring Theory

A Note on Coproper Corings

Jawad Y. Abuihlail
Birzeit University, Palestine
jabuhlail@birzeit.edu

In this note we introduce and study the **coproper measuring** left (right) α-pairings. Our results generalize, to the case of corings over arbitrary ground rings, several results on left (right) coproper coalgebras over base fields.

Identities in nil-algebras over a field of a prime characteristic

Elena Aladova
Moscow Pedagogical State University, Russia.
aladovael@mail.ru
Coauthors: Alexei Krasilnikov

Let $\mathbb{F}$ be a field, let A be a free associative algebra (without 1) over $\mathbb{F}$ on free generators $x_1, x_2, \ldots$ and let G be an associative $\mathbb{F}$-algebra. Let $f(x_1, \ldots, x_n)$ in A. We say that $f(x_1, \ldots, x_n) = 0$ is a *polynomial identity* (or an *identity*) in G if $f(g_1, \ldots, g_n) = 0$ for all $g_1, \ldots, g_n$ in G. Two systems of polynomial identities are *equivalent* if every associative $\mathbb{F}$-algebra satisfying all the identities of the first system satisfies all the identities of the second system and vice versa. If a system of polynomial identities is equivalent to some finite system we say that the system is *finitely based* or *has a finite basis.* If $\mathbb{F}$ is a field of characteristic 0 then every system of polynomial identities in associative $\mathbb{F}$-algebras has a finite basis (Kemer, 1987). On the other hand, if $\mathbb{F}$ is a field of a prime characteristic then there exist non-finitely based systems of polynomial identities (Belov, 1999; Grishin, 1999; Shchigolev, 1999).

In 1999 Grishin constructed a non-finitely based system of polynomial identities in nil-algebras over a field of characteristic 2 which contains the identity $x^{32} = 0$. Similar system with the identity $x^6 = 0$ was constructed by Gupta and Krasilnikov in 2002. Over a field of a prime characteristic p Shchigolev (1999) constructed a non-finitely based system of polynomial identities containing the identity $x^n = 0$, where $n = 2p^3(2p+1)$. In a particular case of $p = 3$ his result was improved by Aladova (2002) who constructed a similar system over a field of characteristic 3 with the identity $x^{12} = 0$ (Shchigolev's system in that case contains the identity $x^{378} = 0$). The aim of the present talk is to improve Shchigolev's result over a field of characteristic $p > 3$.

Let $\mathbb{F}$ be a field of characterictic $p > 3$, let $[x, y] = xy - yx$, $f(x_1, x_2) = x_1^{p-1} x_2^{p-1} [x_1, x_2]$ and let

$$w_n = [[x, y], z] f(x_1, x_2) \cdots f(x_{2n-1}, x_{2n}) [[x', y'], z'] ([[x, y], z][[x', y'], z'])^{p-1}.$$

Our main result is as follows.

Theorem. Over a field $\mathbb{F}$ of characteristic $p > 3$ the system of polynomial identities

$$\{x^{6p} = 0, w_n = 0 \mid n = 0, 1, \dots\}$$

is not equivalent to any finite system of identities in associative $\mathbb{F}$-algebras.

To prove the theorem we construct, for each positive integer n, an associative $\mathbb{F}$-algebra B_n such that B_n satisfies the identities $x^{6p} = 0$ and $w_k = 0$ for all $k <= n$ but it does not satisfy the identity $w_{n+1} = 0$. In the proof we use some results of Shchigolev.

Homological dimensions of Modules over Commutative Noetheian rings

Javad Asadollahi Dehaghi
Shahre-Kord University, Iran
Asadollahi@ipm.ir
Coauthors: Shokrollah Salarian

We extend a criterion of Gerko for a ring to be Cohen-Macaulay to arbitrary, not necessarily local, Noetherian rings. Our version reads as follows: The Noetherian ring R is Cohen-Macaulay if and only if for all finitely generated R-modules M, $CM - dim_R M$ is finite, where CM-dim denotes Cohen-Macaulay dimension.

On the FGI-Rings

Mamadou Barry
Université Cheikh Anta DIOP, Dakar, Senegal
mansabadion1@hotmail.com
Coauthors: Mamadou Sanghare

Let R be a commutative field. An R-vector space V is said to satisfy the property (I) if every injective endomorphism of V is an automorphism of V. In the category of R-vector spaces, property (I) characterizes the finitely generated vector spaces. For any ring R, the property (I) does not characterize the finitely generated R-modules even though R is a commutative ring. For example, the Z-module Q satisfies property (I) and Q, considered as Z-module, is not finitely generated. R is called an FGI-ring if property (I) characterizes the finitely finitely generated R-modules. The purpose of this paper is to characterize the commutative FGI-rings.

Applications of the group identities theory to the group of units

Adalbert Bovdi
University of Debrecen, Hungary
bodibela@math.klte.hu

The theory of the group of units has undergone significant changes in the past 10 years. A new progress in the theory of group identities, achieved by several authors, implies new results in the theory of group of units. Using the theory of the group identities, we have obtained a several new results. For example:

Theorem. Let F be a field of characteristic p, and G be a group having a nontrivial p-Sylow subgroup P. Then $U(FG)$ is n-Engel if and only if G is a nilpotent group with a normal subgroup H such that $[G : H] = p^m$ and the commutator subgroup of H is a finite p-group; consequently FG is n-Engel algebra.

Nonsingular zeros of quartic polynomials over finite fields

Lekbir Chakri
Abdus Salam ICTP, Trieste, Italy
lchakri@ictp.trieste.it

In this talk, we show that if f is an absolutely irreducible quartic polynomial, with order at least 7, and with constant term zero over finite field F_q of odd cardinality $q \geq 37$, then f has a nonsingular F_q-rational zero.

Torsion Classes and F-Noetherian Coalgebras

Juan Cuadra
Universidad de Almeria, Spain
jcdiaz@ual.es

From a ring theoretic point of view, the category of right comodules $\mathcal{M}^C$ over a coalgebra C may be viewed as a pretorsion class in ${}_{C^*}\mathcal{M}$, the category of left modules over the dual algebra C^*. It is known that $\mathcal{M}^C$ is a torsion class in ${}_{C^*}\mathcal{M}$ when the corresponding linear topology on C^* contains a basis of finitely generated ideals. In this talk we will give a partial converse to this result.

Groups of units of integral group rings of Kleinian type

Ángel del Río
Universidad de Murcia, Spain
adelrio@um.es
Coauthors: Antonio Pita, Manuel Ruiz

We show how to use methods on Kleinian groups to the study of integral group rings of finite groups and classify the finite groups for which these methods apply.

Invariants of free algebras

Vitor O. Ferreira
University of Sao Paulo, Brasil
vofer@ime.usp.br
Coauthors: L. S. I. Murakami, A. Paques

A Galois correspondence is exhibited between right coideal subalgebras of a finite-dimensional pointed Hopf algebra acting homogeneously and faithfully on a free associative algebra and free subalgebras containing the invariants of the action.

Primal ideals in commutative rings without finiteness conditions

Laszlo Fuchs
Tulane University, New Orleans, USA
fuchs@tulane.edu

An ideal A of a commutative ring R is called primal if in the factor ring R/A the zero-divisors form an ideal; this ideal must be of the form P/A with P a prime ideal of R. Irreducible ideals are always primal, hence every ideal can be represented as an intersection of primal ideals. The existence of (possibly infinite) irredundant intersections as well as their uniqueness properties will be discussed in general, as well as in special cases, including Noetherian and arithmetical rings, and Pruefer domains.

C-monoids and the multiplicative structure of integral domains

Franz Halter-Koch
Universität Graz, Austria
franz.halterkoch@uni-graz.at
Coauthors: A. Geroldinger and W. Hassler

Let A be an order in a Krull domain R with a non-zero conductor. Then the multiplicative monoid of A is a special congruence monoid and has the structure of a C-monoid, provided that R has finite class group and the residue class ring of the conductor is finite. As C-monoids are arithmetically well-understood auxiliary objects, this fact is basic for an investigation of the factorization properties of A.

Reduced K-theory for Azumaya algebras

Roozbeh Hazrat
Australian National University, Australia
hazrat@maths.anu.edu.au

We attempt to investigate the behaviour of the reduced Whitehead group $K_1(A)$, where A is an Azumaya algebra over a commutative ring R. Begining with the work of Platonov, who answered the Bass-Tannaka-Artin problem ($SK_1(D)$ is not trivial in general) there has been an extensive investigation both on functorial behaviour and computational aspect of this group over the category of the central simple algebras. Here we try to study this group on a bigger category, namely Azumaya Algebras.

About Serre's conjecture in noncommutative case.

Natalia K. Iyudu
Moscow State University, Russia
ioudu@mail.ru
Coauthors: Robert Wisbauer

As shown by D. Quillen and S. Souslin, Serre's conjecture about freeness of finitely generated projective modules has a positive solution for polynomial rings. But, for example, for Weyl algebras or free noncommutative polynomials on two variables over division rings, it is not true. Namely, there exists a stably free non-free module over any ring from these classes. We construct a counterexample to Serre's conjecture for the class of noncommutative algebras given by certain relations which generalize the defining relations of Weyl algebra. In partiqular RIT (relativistic internal time) algebra (which appeared in physics) belongs to this class and the corresponding counterexample was obtained in paper [1] by the autors.

[1] I. Antoniou, N. Iyudu, R.Wisbauer *On Serre's problem for RIT Algebra*, Communications in Algebra, (to appear)

Some extremal varieties of associative algebras

Elena Kireeva
Moscow Pedagogical State University, Russia.
`vvd@cityline.ru`
Coauthors: Alexei Krasilnikov

Let F be a field, A be a free associative algebra over F on free generators $x_1, x_2, \ldots$ An element $v = v(x_1, \ldots, x_n)$ of the algebra A is called a *polynomial identity* or *identity* in an associative F-algebra G if $v(g_1, \ldots, g_n) = 0$ for all elements $g_1, \ldots, g_n$ of the algebra G. In this case an expression $v = 0$ is also called an identity. The class of all associative F-algebras satisfying a given set of identities is called a *variety.* An ideal V in the algebra A is called T-ideal if V is a fully invariant ideal, that is V is closed under all endomorphisms of A. It is known that there is one-to-one correspondence between the set of all varieties of associative F-algebras and the set of all T-ideals in A. Namely if $\mathbf{V}$ is a variety of associative F-algebras then the corresponding T-ideal is the ideal of all identities which are satisfied in every algebra of $\mathbf{V}$ and if V is a T-ideal in A then the corresponding variety is the variety of all F-algebras satisfying every identity of V. The quotient algebra A/V is called a *relatively free algebra* (of countable infinite rank) of the variety $\mathbf{V}$. We refer to [1], [4], [5] and [8] for the terminology and basic facts concerning varieties of associative algebras and polynomial identities.

Let V be a T-ideal in A. A vector subspace U in A/V is called a T-space if U is a fully invariant subspace, that is U is closed under all endomorphisms of the algebra A/V. A T-space U is *finitely generated* if there exist a finite set of elements $u_1, \ldots, u_k$ of U such that every element u of U can be written as a linear combination of the elements $f_1(u_1), \ldots, f_k(u_k)$ for some endomorphisms $f_1, \ldots, f_k$ of the algebra A/V.

Let F be a field of characteristic $p > 0$ and let $\mathbf{V}_p$ be the variety of associative F-algebras given by the identities $[[x, y], z] = 0$ and $x^p = 0$ if $p > 2$, and by the identities $[[x, y], z] = 0$ and $x^4 = 0$ if $p = 2$ (where $[x, y] = xy - yx$). It was proved by A.V. Grishin [6] for $p = 2$ and by V.V. Shchigolev [10] for $p > 2$ that the relatively free algebra of countable infinite rank of the variety $\mathbf{V}_p$ contains non-finitely generated T-spaces. The construction of such T-spaces is one of the most important steps in the solution of the Specht problem over a field of a prime characteristic (see [2], [6], [9] and also [3], [7], [10]).

Our main result is as follows.

Theorem Let F be a field of characteristic $p > 0$. Then $\mathbf{V}_p$ is a minimal variety of associative F-algebras whose relatively free algebras of countable infinite rank contain non-finitely generated T-spaces.

References

[1] Yu.A. Bakhturin and A.Yu. Olshanskii, Identities, *Algebra* II, Encyclopedia of Mathematical Sciences 18, Springer, Berlin, 1991, 107-221.

[2] A.Ya. Belov, On non-Specht varieties, *Fundam. Prikl. Mat.* **5** (1999), 47-66 (Russian).

[3] A.Ya. Belov, Counterexamples to the Specht problem, *Mat.Sb.* **191** (2000), 13-24 (Russian). *Sb. Math.* **191** (2000), 329-340.

[4] L.A. Bokhut, I.V. Lvov and V.K. Kravchenko, Noncommutative rings, *Algebra* II, Encyclopedia of Mathematical Sciences 18, Springer, Berlin, 1991, 1-106.

[5] V. Drensky, *Free algebras and PI-algebras*, Springer, Singapore, 2000.

[6] A.V. Grishin, Examples of T-spaces and T-ideals of characteristic 2 without the finite basis property, *Fundam. Prikl. Mat.* **5** (1999), 101-118 (Russian).

[7] A.V. Grishin, On non-Spechtianness of the variety of associative rings that satisfy the identity $x^{32} = 0$, *Electron. Res. Announc. Amer. Math. Soc.* **6** (2000), 50-51.

[8] L.H. Rowen, *Ring Theory*, Academic Press, Boston, 1991.

[9] V.V. Shchigolev, Examples of infinitely based T-ideals, *Fundam. Prikl. Mat.* **5** (1999), 307-312 (Russian).

[10] V.V Shchigolev, Examples of infinitely basable T-spaces, *Mat.Sb.* **191** (2000), 143-160 (Russian). *Sb. Math.* **191** (2000), 459-476.

Partially supported by INTAS.

An algorithm for the embedding of the given p-group into the normalised unit group of the modular group algebra of a finite p-group

Alexander Konovalov

Zaporozhye State University, Ukraine

konovalov@member.ams.org

Coauthors: Wolfgang Kimmerle

Let p be a prime number, G be a finite p-group and K be a field of p elements. Let I is the augmentation ideal of the modular group algebra KG. Then the normalised unit group $V(KG)$ consists of all elements of the type $1 + x$, where $x \in I$.

We consider the following algorithmic questions:

1. Decide whether for p-groups G and H their group algebras KG and KH are isomorphic;
2. Find all subgroups B of $V(KG)$, such that B is isomorphic to the given finite p-group H;
3. Find all subgroups B of $V(KG)$, such that B is isomorphic to the given finite p-group H and elements of B are linearly independent in KG (in case $G = H$ — find all group bases B of KG, such that B is isomorphic to G).

We present an algorithm to solve this problems, constructing the required embedding of the given group into the group algebra or deciding that this is impossible. It was implemented with the help of computer algebra system GAP [4] using the package LAGUNA [1] which extends GAP functionality for calculations in group rings and allows construction and efficient calculations in the normalised unit group. The algorithm will be included into the next release of LAGUNA.

Note that the first problem could be also using the program SISYPHOS [5], which was only partially compatible with the GAP system and does not use calculations in the unit group. We provided another algorithm, fully compatible with the GAP system. We used SISYPHOS output for double-checking results while testing the program.

As an example of this algorithm usage, we investigated the question by Shalev, asking whether $V(KG)$ possesses a section isomorphic to the wreath product of a cyclic group of order p and the derived subgroup of G [3]. Shalev proved that this is true for the case of odd p and a cyclic derived subgroup of G. In [2] the positive answer was given for 2-groups of maximal class, constructing the required wreath product as a section. It is an interesting question, whether such wreath product could be constructed in $V(KG)$ as a subgroup. Using our algorithm, we did not discovered such examples among 2-groups of maximal class.

References

[1] V. Bovdi, A. Konovalov, R. Rossmanith, C. Schneider. *LAGUNA — Lie AlGebras and UNits of group Algebras, Version 3.0*, 2003 (`http://ukrgap.exponenta.ru/laguna.htm`).

[2] A. Konovalov. Wreath products in the unit group of modular group algebras of 2-groups of maximal class. *Acta Math. Acad. Paed. Nyíregyháziensis* **17** (2001), 141-149. (`http://elib.uni-osnabrueck.de/EMIS/journals/AMAPN/vol17_3/1.html`).

[3] A. Shalev. On some conjectures concerning units in p-group algebras. Proceedings of the Second International Group Theory Conference (Bressanone, 1989). *Rend. Circ. Mat. Palermo (2)* **23** (1990), 279-288.

[4] The GAP Group, *GAP — Groups, Algorithms, and Programming, Version 4.2*, Aachen, St Andrews, 1999 (`http://www.gap-system.org`).

[5] M. Wursthorn. Isomorphisms of modular group algebras: an algorithm and its application to groups of order 2^6. *J. Symbolic Comput.* **15** (1993), 211-227.

The second author was supported by DAAD grant A0216278/Ref.322

Expansions of prime ideals in noncommutative rings

Peter Malcolmson
Wayne State University, Detroit, USA
`petem@math.wayne.edu`
Coauthors: Frank Okoh

A commutative integral domain R is said to have the *Roquette-Samuel property* if the unit group of any finitely generated commutative extension domain S of R is finitely generated over the unit group of R. If only finitely many prime elements of R can become units in any such S, then we say that R is *robust.*

Makar-Limanov has suggested that it may be possible to specialize skew fields to most finite characteristics. In trying to obtain an invariant of skew fields according to that program, we extend the above notions to the noncommutative setting. Unlike the commutative case we show that the ring of rational integers does not have the Roquette-Samuel property nor is it robust.

Pseudo-chain rings and pseudo-uniserial modules

Ryszard Mazurek
Technical University of Bialystok, Poland
`mazurek@pb.bialystok.pl`

A ring R is called a right pseudo-chain ring if for any prime right ideal P and elements a, b of R, aP and bR are comparable under inclusion. These rings are a common generalization of right chain rings and commutative pseudo-valuation rings. We give several characterizations of right pseudo-chain rings and consider the stability of these rings with respect to homomorphic images, localizations, and under passage to overrings. We also introduce the concept of a pseudo-uniserial module. The class of pseudo-uniserial modules contains all uniserial modules and it is closed with respect to submodules and homomorphic images.

The exchange property of Gromov's translation algebras

Francesc Perera
Queen's University, Belfast, U.K.
`perera@qub.ac.uk`
Coauthors: Pere Ara, Kevin C. O'Meara

In this talk we shall introduce a new source of examples of exchange rings, in the form of certain Gromov translation rings over von Neumann regular rings. As a consequence of our results, it will be shown that the growth algebras introduced by the Hannah and O'Meara in 1993 are exchange rings. This provides, for any prescribed value r in $[0, 1]$, an (even countable dimensional) exchange algebra with bandwidth dimension r.

On a question of Mueller

Phạm Ngọc Ánh
Álfréd Rényi Mathematical Institute, Budapest, Hungary
anh@renyi.hu
Coauthors: D. Herbera

A positive solution is given to a question of B.J. Mueller on the existence of an automorphism of a semiperfect complete HNP-ring inducing a good duality.

On one-accessible regular algebras

Robert Raphael
Concordia University, Montreal, Canada
raphael@alcor.concordia.ca
Coauthors: K. Beidar, K. O'Meara

We study regular algebras R over a field F such that for any $a \in R$ there exist $0 \neq b \in F[a]$ and $b' \in R$ such that $bb'b = b$, $b'bb' = b'$ and the subalgebra of R generated by a and b' is regular. Such algebras are called one-accessible. We show that a finite product of matrix rings over a field is one-accessible and that a regular algebra over an uncountable perfect field is one-accessible if and only if it is algebraic. We elucidate and characterize when a nilpotent element has all its powers regular (or unit-regular) in an arbitrary algebra R over a commutative ring Λ. This involves finite direct products of matrix rings over factor rings of Λ.

On lattice-ordered commutative semigroups

Raquel Reis
Universidade Aberta, Portugal
raqreis@univ-ab.pt
Coauthors: Laszlo Fuchs

W. Krull: "Several results in the ideal theory of commutative rings do not depend on the fact that the ideals are composed ofelements: they are valid in lattice-ordered monoids."

Most papers in "abstract ideal theory" assume the maximum condition, however, we work without the finiteness condition.

Our main concern: representations of elements as intersections of associated components with special properties.

On a theorem of Ian Hughes about division rings of fractions

Javier Sánchez Serdà
Universitat Autònoma de Barcelona, Spain
jsanchez@mat.uab.es
Coauthors: Warren Dicks, Dolors Herbera

We say a group G is *locally indicable* if every non-trivial finitely generated subgroup of G has the infinite cyclic group as homomorphic image. Thus free groups, locally free groups, and torsion-free abelian groups are locally indicable. Other important examples are ordered groups [4] and torsion-free one-relator groups [1] [2].

Let G be a locally indicable group, k a division ring, and kG a crossed-product group ring. Suppose kG has a division ring of fractions D. For every subgroup H of G we denote by $D(H)$ the division ring of fractions of kH inside D. We say D is a *Hughes-free division ring of fractions* of kG if for every nontrivial finitely generated subgroup T of G, and every normal subgroup H of T such that T/H is infinite cyclic, there exists t in T such that Ht generates T/H and t is $D(H)$ linearly independent.

Ian Hughes [3] proved that, up to kG-isomorphism, at most one division ring of fractions of kG is Hughes-free. We simplify the proof of the theorem and introduce concepts that illuminate Hughes' arguments.

Hughes' result was applied by others in work on division rings of fractions of group rings of free groups.

References

[1] S. D. Bodskii, *Equations over groups and groups with one defining relation*, Sibirsk. Mat. Zh. **25(2)** (1984), 84-103; Siberian Math. J. **25(2)** (1984), 235-251.

[2] J. Howie, *On locally indicable groups*, Math. Z. **180(4)** (1982), 445-461.

[3] I. Hughes, *Division rings of fractions for group rings*, Comm. Pure Appl. Math. **23** (1970), 181-188.

[4] F. Levi, *Contributions to the theory of ordered groups*, Proc. Indian Acad. Sci. Sect. A. **17** (1943), 199-201.

On rings with the same set of proper ideals

Hisa Tsutsui
Millersville University, USA
Dr.T@math-sci.net
Coauthors: Yasuyuki Hirano

We investigate rings whose set of proper ideals are identical.

Structure theorems on countably compact rings

Mihail Ursul
University of Oradea, Romania
ursul@uoradea.ro

We give a new proof of Wedderburn Theorem for compact rings of prime characteristic. An example of a countably compact ring of prime characteristic whose Jacobson radical is not a semidirect factor is given. It is proved that the Jacobson radical of a countably compact ring is the intersection of all maximal topologically nilpotent subrings.

The matrix type of rings

Peter Vamos
University of Exeter, UK
p.vamos@ex.ac.uk

There are some 'pathological' rings e.g. when the ring of 2×2 matrices are isomorphic to that of 6×6 matrices. Of course beauty is in the eye of the opportunistic researcher. In this talk we'll look at these strangely alluring rings.

The matrix type of a ring R is defined to be the binary relation on $\mathbb{N}$ where $m, n \in \mathbb{N}$ are related if the $m \times m$ matrix ring $M_m(R)$ is isomorphic (as a ring) to the $n \times n$ matrix ring $M_n(R)$. This is a multiplicative congruence on $\mathbb{N}$. If this the equality then we say that R has the Invariant Matrix Number (**IMN**) property.

We will review the properties of the matrix type and seek examples of rings with a given type. Connections to the basis type (analogous to matrix type but recording the pairs (m, n) for which R^n is isomorphic to R^m as R-modules) and to the Cuntz C^* algebras will be investigated. We will also find conditions for a ring to have the **IMN** property and relate this to its analogue in basis type, the **IBN** property.

Intersections of powers of a principal ideal and primality

Paolo Zanardo
University of Padova, Italy
pzanardo@math.unipd.it

We consider the following natural property: we say that an integral domain R satisfies property (*) if, for every non-unit a of R, the intersection of the powers a^nR is a prime ideal. We study property (*) for integral domains R which arise in a classical way. Namely, R is the integral closure of a valuation domain V in a finite algebraic extension L of the field of fractions Q of V. The degree of the extension makes a crucial difference in the results.

Classes of modules over regular semiartinian rings

Jan Zemlicka
Karlova Univ., Praha, Czech Republic
zemlicka@karlin.mff.cuni.cz

A ring R is said to be right semiartinian if the socle of every non-zero cyclic right R-module is non-zero. Let R be a von Neumann regular right semiartinian ring with primitive factors artinian. Then R is left semiartinian as well, and it can be described by the dimension sequence, a suitable sequence of ordinals and skew-fields. We will show correspondence between the structure of R (in particular the dimension sequence) and some classes of right modules such as the class of all dually slender modules or the class of all tilting modules.

Standard Noether Normalizations of the Graph Subring

Rita Zuazua
Universidad Nacional Autónoma de Mexico, Morelia, Mexico
zuazua@matmor.unam.mx
Coauthors: Florian Luca

Let G be a simple graph with n vertices. We write $E(G)$ for the set of edges of G, and m for the cardinality of $E(G)$. We let $\mathbf{k}$ be a field of characteristic zero, and we write $R := \mathbf{k}[x_1, \ldots, x_n]$ for the standard polynomial ring in n indeterminates over $\mathbf{k}$. The *monomial subring* or the *edge subring* of G (over $\mathbf{k}$) is the subring

$$\mathbf{k}[G] := \mathbf{k}[\{x_i x_j \mid \{i,j\} \in E(G)\}] \subset R. \tag{1}$$

The *presentation ideal* $P(G)$ of $\mathbf{k}[G]$ is simply the kernel of the epimorphism of $\mathbf{k}$-algebras

$$f : S = \mathbf{k}[\{t_e \mid e \in E(G)\}] \longrightarrow \mathbf{k}[G], \tag{2}$$

induced by $f(t_e) = x_i x_j$ when $e := \{i,j\} \in E(G)$, where $\{t_e\}$ is a new set of variables in one to one correspondence with the edges of G. Notationally, we shall find it convenient to refer to the indeterminates t_e for $e \in E(G)$ as t_{ij}, for i diferent to j, with the convention that t_{ij} is defined precisely when i is incident to j, and in this case we do not distinguish between t_{ij} and t_{ji}.

A *Noether normalization* of $\mathbf{k}[G]$, or, equivalently, of $S/P(G)$, is an integral extension of the form

$$\mathbf{k}[h_1, \ldots, h_d] \longrightarrow S/P(G), \tag{3}$$

where $h_1, \ldots, h_d$ are homogeneous polynomials in the t_{ij}'s and d is the Krull dimension of $\mathbf{k}[G]$. When all the h_ℓ's for $\ell = 1, \ldots, d$ are linear forms in the indeterminates t_{ij}, then the Noether normalization is called *standard*.

In [1], Alcántar asked if a standard Noether normalization of $\mathbf{k}[G]$ of the form

$$h_i := \sum_j a_{ij} t_{ij} \quad \text{with } a_{ij} \in \mathbf{k}, \quad i = 1, \ldots, n \tag{4}$$

exists provided that $\mathbf{k}[G]$ has Krull dimension n.

We will give an affirmative answer to this question.

References

[1] A. Alcántar, *Noether normalizations of some subgrings of graphs*, Comm. Alg. **29** (2001), 5525–5534.